

BPS black holes in N=2 D=4 gauged supergravities

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Abstract

We construct and analyze BPS black hole solutions in gauged $N = 2, D = 4$ supergravity with charged hypermultiplets. A class of solutions can be found through spontaneous symmetry breaking in vacua that preserve maximal supersymmetry. The resulting black holes do not carry any hair for the scalars. We demonstrate this with explicit examples of both asymptotically flat and anti-de Sitter black holes.

Next, we analyze the BPS conditions for asymptotically flat black holes with scalar hair and spherical or axial symmetry. We find solutions only in cases when the metric contains ripples and the vector multiplet scalars become ghost-like. We give explicit examples that can be analyzed numerically. Finally, we comment on a way to circumvent the ghost-problem by introducing also fermionic hair.

1 Introduction

The main aim of this paper is the search for supersymmetric four-dimensional black holes in gauged $N = 2$ supergravities in the presence of hypermultiplets, charged under an abelian gauge group. In the original references on BPS black holes in $D = 4, N = 2$ supergravity [1, 2, 3, 4, 5], and subsequent literature, see e.g. [6, 7, 8, 9], one usually considers ungauged hypermultiplets, which then decouple from the supersymmetry variations and equations of motion for the vector multiplet fields. We want to explore how the story changes when the hypers couple non-trivially to the vector multiplets via gauge couplings and scalar potentials that are allowed within gauged $N = 2$ supergravity [10, 11, 12, 13, 14]. For the simpler case of minimally gauged supergravity, where no hypermultiplets are present but only a cosmological constant or Fayet-Iliopoulos terms, asymptotically anti-de Sitter BPS black holes can be found. This has been discussed in the literature, starting from the early references [15, 16], or more recently in [17]. We initiate here the extension to general $D = 4, N = 2$ gauged supergravities, including hypermultiplets.

One of our motivations comes from understanding the microscopic entropy of (asymptotically flat) black holes. In ungauged supergravity, arising e.g. from Calabi-Yau compactifications, this is relatively well understood in terms of counting states in a weakly coupled D-brane set-up [18, 19], and then extrapolating from weak to strong string coupling. In flux compactifications, with effective gauged supergravity actions, this picture is expected to be modified. The most dramatic modification is probably when the dilaton is stabilized by the fluxes, such that one cannot extrapolate between strong and weak string coupling.

Another motivation stems from the AdS/CFT correspondence and its applications to strongly coupled field theories. Here, finite temperature black holes that asymptote to anti-de Sitter space-time describe the thermal behavior of the dual field theory. Often, like e.g. in holographic superconductors, see e.g. [20] for some reviews or [21] for more recent work, charged scalar fields are present in this black hole geometry, providing non-trivial scalar hair¹ that can be computed numerically. Therefore, one is in need to find large classes of asymptotically AdS black holes with charged scalars. This is one of the aims of this paper. Although we mostly work in the context of supersymmetric black holes, some of our analysis in section 3 can be carried out for finite temperature black holes as well.

¹By scalar hair, in this paper, we mean a scalar field that is zero at the horizon of the black hole, but non-zero outside of the horizon. According to this definition, the vector multiplet scalars subject to the attractor mechanism in $N = 2$ ungauged supergravity, do not form black holes with scalar hair. The solutions that we discuss in section 5, however, will have hair.

The plan of the paper is as follows. First, in section 2, we give a brief summary of the known black hole solutions in $N = 2$ supergravity with neutral hypermultiplets, making a clear distinction between the asymptotically flat and asymptotically AdS space-times. We then explain the model with gauged hypermultiplets we are interested in and how this fits within the framework of $N = 2$ gauged supergravity.

In section 3 we first explain how one can use a Higgs mechanism for spontaneous gauge symmetry breaking, in order to obtain effective $N = 2$ ungauged theories from a general gauged $N = 2$ supergravity. We keep the discussion short since these results follow easily from our previous paper [22]. Then we show how this method can be used to embed already known black hole solutions into gauged supergravities and explain the physical meaning of the new solutions. We illustrate this with an explicit example of a static, asymptotically flat black hole with the well-known STU model and one gauged hypermultiplet (the universal hypermultiplet). We also give examples of AdS black holes with charged scalars, that may have applications in the emerging field of holographic superconductivity [20, 21].

In section 4 we discuss in more general terms asymptotically flat, stationary spacetimes preserving half of the supersymmetries. We analyze the fermion susy variations in gauged supergravity after choosing a particular ansatz for the Killing spinor. One finds two separate cases, defined by $T_{\mu\nu}^- = 0$ and $P_\Lambda^x = 0$, respectively. Whereas the former case contains only Minkowski and AdS₄ solutions, the latter leads to a class of solutions that generalize the standard black hole solutions of ungauged supergravity. We analyze this in full detail in section 4.6.2 and give the complete set of equations that guarantees a half-BPS solution. We then explain how this fits to the solutions obtained in section 3.

Finally, in section 5, we study asymptotically flat black holes with scalar hair. We find two separate classes of such solutions. One is a purely bosonic solution with scalar hair, but with the shortcoming of having ghost modes in the theory. The other class of solutions has no ghosts but along with scalar hair we also find fermionic hair, i.e. the fermions are not vanishing in such a vacuum.

Some of the more technical aspects of this paper, including explicit hypermultiplet gaugings, are presented in the appendices.

2 Preliminaries

In the first part of this section, we set our notation and briefly review the BPS black hole solutions in four-dimensional ungauged $N = 2$ supergravity. In the second part, we present the (bosonic) action for $N = 2$ supergravity coupled to charged hypermultiplets with abelian gaugings, and review some of the BPS black holes that asymptote to anti-de Sitter spacetime. For a review of $N = 2$ (gauged) supergravity we refer to [13], which notation we closely follow.

2.1 Ungauged supergravity

We start by discussing the $N = 2$ Lagrangian for ungauged supergravity coupled to (abelian) vector and hypermultiplets. The scalar fields in both these multiplets are in this case all neutral. The theory has an action $S = \int d^4x \sqrt{-g} \mathcal{L}$, and the bosonic part of the Lagrangian \mathcal{L} is given by

$$\mathcal{L} = \frac{1}{2}R(g) + g_{i\bar{j}}\partial^\mu z^i \partial_\mu \bar{z}^{\bar{j}} + h_{uv}\partial^\mu q^u \partial_\mu q^v + I_{\Lambda\Sigma}(z)F_{\mu\nu}^\Lambda F^{\Sigma\mu\nu} + \frac{1}{2}R_{\Lambda\Sigma}(z)\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^\Lambda F_{\rho\sigma}^\Sigma. \quad (2.1)$$

We keep the same convention for metric signatures and field strengths as in [22]. In particular, the spacetime metric has signature $(+---)$, and we work in units in which the gravitational coupling constant is set to one, $\kappa^2 = 1$.² The z^i ($i = 1, \dots, n_V$) are the complex scalars in the vector multiplets, with special Kähler metric $g_{i\bar{j}}(z, \bar{z})$. This geometry is best described in terms of holomorphic sections $X^\Lambda(z)$ and $F_\Lambda(z)$, $\Lambda = 0, 1, \dots, n_V$, such that the Kähler potential takes the form

$$\mathcal{K}(z, \bar{z}) = -\ln \left[i(\bar{X}^\Lambda(\bar{z})F_\Lambda(z) - X^\Lambda(z)\bar{F}_\Lambda(\bar{z})) \right]. \quad (2.2)$$

When a prepotential exists, it is given by $2F = X^\Lambda F_\Lambda$. It should be homogeneous of second degree, and one must have that $F_\Lambda(X) = \partial F(X)/\partial X^\Lambda$. Our general analysis does not assume the existence of a prepotential. The complex conjugate of the “period-matrix” $\mathcal{N}_{\Lambda\Sigma}$ is defined by the matrix multiplication

$$\bar{\mathcal{N}}_{\Lambda\Sigma} \equiv \begin{pmatrix} D_i F_\Lambda \\ \bar{F}_\Lambda \end{pmatrix} \cdot \begin{pmatrix} D_i X^\Sigma \\ \bar{X}^\Sigma \end{pmatrix}^{-1}, \quad (2.3)$$

with $\mathcal{K}_i = \partial_i \mathcal{K}$, $D_i X^\Lambda = (\partial_i + \mathcal{K}_i)X^\Lambda$, and similarly $D_i F_\Lambda = (\partial_i + \mathcal{K}_i)F_\Lambda$. Their imaginary and real parts are denoted by

$$R_{\Lambda\Sigma} \equiv \text{Re} \mathcal{N}_{\Lambda\Sigma}, \quad I_{\Lambda\Sigma} \equiv \text{Im} \mathcal{N}_{\Lambda\Sigma}. \quad (2.4)$$

²We corrected the sign in front of the Einstein-Hilbert term compared to the first version in [22].

The scalars in the hypermultiplet sector parametrize a quaternion-Kähler manifold, whose metric can be expressed in terms of quaternionic vielbeine. In local coordinates q^u ; $u = 1, \dots, 4n_H$, we have

$$h_{uv}(q) = \mathcal{U}_u^{A\alpha}(q) \mathcal{U}_v^{B\beta}(q) \mathbb{C}_{\alpha\beta} \epsilon_{AB} , \quad (2.5)$$

where $\mathbb{C}_{\alpha\beta}$, $\alpha, \beta = 1, \dots, 2n_H$ and ϵ_{AB} , $A, B = 1, 2$ are the antisymmetric symplectic and $SU(2)$ metrics, respectively. The value of the Ricci-scalar curvature of the quaternionic metric is always negative and fixed in terms of Newton's coupling constant κ . In units in which $\kappa^2 = 1$, which we will use in the remainder of this paper, we have

$$R(h) = -8n_H(n_H + 2) . \quad (2.6)$$

We will discuss more on the quaternionic geometry when we introduce the gauging at the end of this section. Clearly, the hypermultiplet scalars q^u do not mix with the other fields (apart from the graviton) at the level of the equations of motion, and it is therefore consistent to set them to a constant value.

2.2 Black holes in asymptotically Minkowski spacetime

Asymptotically flat and stationary BPS black hole solutions of ungauged supergravity have been a very fruitful field of research in the last decades. In absence of vector multiplets ($n_V = 0$), with only the graviphoton present, the supersymmetric solution is just the well-known extremal Reissner-Nordström (RN) black hole. This solution was later generalized to include a number of vector multiplets [1]. The most general classification of the BPS solutions, including multicentered black holes, was given by Behrndt, Lüst and Sabra [5] and we will refer to those as BLS solutions. We will briefly list the main points of the solutions, as they will play an important role in what follows.

To characterize the black hole solutions, we first denote the imaginary parts of the holomorphic sections by

$$\tilde{H}^\Lambda \equiv i(X^\Lambda - \bar{X}^\Lambda) , \quad H_\Lambda \equiv i(F_\Lambda - \bar{F}_\Lambda) . \quad (2.7)$$

We assume stationary solutions with axial symmetry parametrized by an angular coordinate φ . The result of the BPS analysis is that the metric takes the form³

$$ds^2 = e^K (dt + \omega_\varphi d\varphi)^2 - e^{-K} (dr^2 + r^2 d\Omega_2^2) , \quad (2.8)$$

³Note that all the results are in spherical coordinates, see [5, 9] for the coordinate independent results.

where \mathcal{K} is the Kähler potential of special geometry, defined by

$$e^{-\mathcal{K}} = i (\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda) . \quad (2.9)$$

The metric components and the symplectic vector $(\tilde{H}^\Lambda, H_\Lambda)$ only depend on the radial variable r and the second angular coordinate θ , and the BPS conditions imply the differential equations on ω_φ

$$\frac{1}{r^2 \sin \theta} \partial_\theta \omega_\varphi = H_\Lambda \partial_r \tilde{H}^\Lambda - \tilde{H}^\Lambda \partial_r H_\Lambda , \quad -\frac{1}{\sin \theta} \partial_r \omega_\varphi = H_\Lambda \partial_\theta \tilde{H}^\Lambda - \tilde{H}^\Lambda \partial_\theta H_\Lambda . \quad (2.10)$$

From this follows the integrability condition $H_\Lambda \square \tilde{H}^\Lambda - \tilde{H}^\Lambda \square H_\Lambda = 0$, where \square is the 3-dimensional Laplacian.

What is left to specify are the gauge field strengths $F_{\mu\nu}^\Lambda$. First we define the magnetic field strengths

$$G_{\Lambda\mu\nu} \equiv R_{\Lambda\Sigma} F_{\mu\nu}^\Sigma - \frac{1}{2} I_{\Lambda\Sigma} \epsilon_{\mu\nu\gamma\delta} F^{\Sigma\gamma\delta} , \quad (2.11)$$

such that the Maxwell equations and Bianchi identities take the simple form

$$\epsilon^{\mu\nu\rho\sigma} \partial_\nu G_{\Lambda\rho\sigma} = 0, \quad \epsilon^{\mu\nu\rho\sigma} \partial_\nu F_{\rho\sigma}^\Lambda = 0 , \quad (2.12)$$

such that (F^Λ, G_Λ) transforms as a vector under electric-magnetic duality transformations.

For the full solution it is enough to specify half of the components of F^Λ and G_Λ , since the other half can be found from (2.11). In spherical coordinates, the BPS equations imply the non-vanishing components⁴

$$F_{r\varphi}^\Lambda = \frac{-r^2 \sin \theta}{2} \partial_\theta \tilde{H}^\Lambda , \quad F_{\theta\varphi}^\Lambda = \frac{r^2 \sin \theta}{2} \partial_r \tilde{H}^\Lambda , \quad (2.13)$$

and

$$G_{\Lambda r\varphi} = \frac{-r^2 \sin \theta}{2} \partial_\theta H_\Lambda , \quad G_{\Lambda \theta\varphi} = \frac{r^2 \sin \theta}{2} \partial_r H_\Lambda . \quad (2.14)$$

From (2.12) it now follows that H_Λ and \tilde{H}^Λ are harmonic functions. With the above identities we can always find the vector multiplet scalars z^i , given that we know explicitly how they are defined in terms of the sections X^Λ and F_Λ . The integration constants of the harmonic functions specify the asymptotic behavior of the fields at the black hole horizon(s) (the constants can seen to be the black hole electric and magnetic charges) and at spatial infinity.

⁴The BPS conditions also imply $F_{r\theta}^\Lambda = G_{\Lambda r\theta} = 0$ due to axial symmetry.

The complete proof that these are indeed all the supersymmetric black hole solutions with abelian vector multiplets and no cosmological constant was given in [9]. Note that the BLS solutions describe half-BPS stationary spacetimes with (only for the multi-centered cases) or without angular momentum. The near-horizon geometry around each center is always $AdS_2 \times S^2$ with equal radii of the two spaces, determined by the charges of the black hole. All solutions exhibit the so-called attractor mechanism [1]. This means that the (vector multiplet) scalar fields get attracted to constant values at the horizon of the black hole that only depend on the black hole charges. As the scalars can be arbitrary constants at infinity we also find the so-called attractor flow, i.e. the scalars flow from their asymptotic value to the fixed constant at the horizon. This phenomenon seems not to be related with supersymmetry, but rather with extremality, since attractor mechanisms have been discovered also in non-supersymmetric (but extremal) solutions. The full classification of non-BPS solutions and attractors is, however, more involved and is still in progress.

2.3 Gauged supergravity

We now turn to the bosonic Lagrangian for gauged $N = 2$ supergravity in presence of n_V abelian vector multiplets and n_H hypermultiplets, charged under the abelian gauge group (see e.g. [13] for further explanation and notation). The effect of the gauging is to covariantize the derivatives for the hypermultiplet scalars⁵, and to add a scalar potential:

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}R(g) + g_{i\bar{j}}\partial^\mu z^i \partial_\mu \bar{z}^{\bar{j}} + h_{uv}\nabla^\mu q^u \nabla_\mu q^v \\ & + I_{\Lambda\Sigma}F_{\mu\nu}^\Lambda F^{\Sigma\mu\nu} + \frac{1}{2}R_{\Lambda\Sigma}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^\Lambda F_{\rho\sigma}^\Sigma - g^2V(z, \bar{z}, q) . \end{aligned} \quad (2.15)$$

The covariant derivative $\nabla_\mu q^u \equiv \partial_\mu q^u + g\tilde{k}_\Lambda^u A_\mu^\Lambda$ defines the gauging of some (abelian) isometries of the quaternionic manifold with Killing vectors \tilde{k}_Λ^u and coupling constant g . The scalar potential is given in terms of the Killing vectors and the corresponding triplet of quaternionic moment maps P_Λ^x (see e.g. [13] for more explanation):

$$V = 4h_{uv}\tilde{k}_\Lambda^u \tilde{k}_\Sigma^v \bar{L}^\Lambda L^\Sigma + (g^{i\bar{j}}f_i^\Lambda \bar{f}_{\bar{j}}^\Sigma - 3\bar{L}^\Lambda L^\Sigma)P_\Lambda^x P_\Sigma^x , \quad (2.16)$$

where

$$L^\Lambda = e^{\mathcal{K}/2} X^\Lambda , \quad f_i^\Lambda = e^{\mathcal{K}/2} D_i X^\Lambda . \quad (2.17)$$

⁵For abelian gaugings, the covariant derivative on the vector multiplet scalars is the flat derivative, because the sections $X^\Lambda(z)$ transform in the adjoint representation of the gauge group. One can of course consider non-abelian gaugings, but this would complicate our black hole analysis in subsequent sections. We leave this as a possible generalization for future work.

The action is invariant under the following supersymmetry variations (up to higher order terms in fermions):

$$\delta_\varepsilon \lambda^{iA} = i\partial_\mu z^i \gamma^\mu \varepsilon^A + G_{\mu\nu}^{-i} \gamma^{\mu\nu} \epsilon^{AB} \varepsilon_B + igg^{i\bar{j}} \bar{f}_{\bar{j}}^\Lambda P_\Lambda^x \sigma_x^{AB} \varepsilon_B , \quad (2.18)$$

$$\delta_\varepsilon \zeta_\alpha = i\mathcal{U}_u^{B\beta} \nabla_\mu q^u \gamma^\mu \varepsilon^A \epsilon_{AB} \mathbb{C}_{\alpha\beta} + 2g\mathcal{U}_{\alpha u}^A \tilde{k}_\Lambda^u \bar{L}^\Lambda \varepsilon_A , \quad (2.19)$$

$$\delta_\varepsilon \psi_{\mu A} = \nabla_\mu \varepsilon_A + T_{\mu\nu}^- \gamma^\nu \epsilon_{AB} \varepsilon^B + igS_{AB} \gamma_\mu \varepsilon^B , \quad (2.20)$$

where λ^{iA} , ζ_α and $\psi_{\mu A}$ are the gauginos, hyperinos and gravitinos respectively. We have used the gravitino field strength and mass matrix

$$T_{\mu\nu}^- \equiv 2iF_{\mu\nu}^{\Lambda-} I_{\Lambda\Sigma} L^\Sigma , \quad S_{AB} \equiv \frac{i}{2}(\sigma_x)_{AB} P_\Lambda^x L^\Lambda , \quad (2.21)$$

and the decomposition identity

$$F_{\mu\nu}^{\Lambda-} = i\bar{L}^\Lambda T_{\mu\nu}^- + 2f_i^\Lambda G_{\mu\nu}^{i-} . \quad (2.22)$$

The upper index “ $-$ ” denotes the anti-selfdual part of the field strengths, and in Minkowski spacetime it is complex. The selfdual part is then obtained by complex conjugation. More details are given in appendix A. Details on the supercovariant derivative $\nabla_\mu \varepsilon_A$, that appears in the supersymmetry transformation rules of the gravitinos, are in appendix B.

The fully $N = 2$ supersymmetric configurations obtained from (2.18)-(2.20) were analyzed in [22]. Two possibilities arise, namely for zero or nonzero cosmological constant in the vacuum. For zero cosmological constant, the different supersymmetric spacetimes are either Minkowski or $AdS_2 \times S^2$ (or its Penrose limit, the supersymmetric pp-wave), whereas for nonzero cosmological constant only AdS_4 can be fully BPS. In the former case, additional constraints arise on the scalar fields, namely (for abelian gaugings)

$$\tilde{k}_\Lambda^u L^\Lambda = 0 , \quad P_\Lambda^x = 0 , \quad (2.23)$$

together with $F_{\mu\nu}^\Lambda = 0$ (Minkowski) and $\tilde{k}_\Lambda^u F_{\mu\nu}^\Lambda = 0$ ($AdS_2 \times S^2$). In the latter case, for AdS_4 , one has the conditions

$$\tilde{k}_\Lambda^u L^\Lambda = 0 , \quad P_\Lambda^x f_i^\Lambda = 0 , \quad \epsilon^{xyz} P^y \bar{P}^z = 0 , \quad (2.24)$$

with vanishing field strengths, $F_{\mu\nu}^\Lambda = 0$, and negative scalar curvature for AdS_4 spacetime, $R = -12g^2 P^x \bar{P}^x$, where $P^x \equiv P_\Lambda^x L^\Lambda$. In all these cases, the scalars are constant or covariantly constant. The fully supersymmetric configurations will play an important role in the construction of 1/2 BPS black hole solutions, since both their near horizon and asymptotic region fall into this class. We will discuss this in detail in the following sections.

A particular class of supergravities arises in the absence of hypermultiplets. This situation is interesting, since it allows for a bare negative cosmological constant in the Lagrangian through the moment maps P_Λ^x that appear in the scalar potential. It is well-known that, for $n_H = 0$ and abelian gauge groups, these moment maps can be replaced by constants (similar to Fayet-Iliopoulos terms), giving rise to a potential

$$V = (g^{i\bar{j}} f_i^\Lambda \bar{f}_{\bar{j}}^\Sigma - 3\bar{L}^\Lambda L^\Sigma) P_\Lambda^x P_\Sigma^x, \quad (2.25)$$

with P_Λ^x numerical constants. When also $n_V = 0$, one can take the sections L^Λ to be constants as well, such that the potential is negative and given by $V = -\Lambda$, with $\Lambda = 3P^x \bar{P}^x$.

2.4 Asymptotically AdS_4 black holes with $n_H = 0$

The construction of BPS black holes in AdS_4 spacetimes is technically more involved due to the presence of the gauged hypermultiplets, and at present there is no complete analysis for this case. Until now, only the case with no hypermultiplets, $n_H = 0$, but with a bare cosmological constant or a potential of the type (2.25) has been investigated in the literature [16, 23, 17]. Static and spherically symmetric (non-rotating) black hole solutions preserving some supersymmetry have been constructed, but they seem to suffer from naked singularities [15, 24]. Recent developments however show a way to construct smooth solutions [17]. On the other hand there are proper BPS black holes when one allows for a non-zero angular momentum [16, 25]. The non-BPS and non-extremal solutions, however, do allow for proper horizons also in the non-rotating case.

Let us illustrate some of these issues in the case of static spacetimes in gauged supergravities with no vector multiplets, so there is only a single gauge field, the graviphoton. Here we have the AdS generalization of the Reissner-Nordström black holes (RNAdS). More explicitly, the metric in our signature is

$$ds^2 = V dt^2 - \frac{dr^2}{V} - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (2.26)$$

with

$$V(r) = 1 - \frac{2M}{r} + \frac{Q^2 + P^2}{r^2} - \frac{\Lambda r^2}{3}. \quad (2.27)$$

Here, Λ is the (negative) cosmological constant and Q and P are the electric and magnetic charge respectively. The field strengths are given by

$$F_{tr}^- = \frac{1}{2r^2} (Q - iP), \quad F_{\theta\varphi}^- = \frac{\sin\theta}{2} (P + iQ). \quad (2.28)$$

For the 1/2 BPS solution the magnetic charge is vanishing, $P = 0$ and $M = Q$ [15]. Of course, this example describes naked singularities rather than black holes. This is because $V(r)$ has no zeroes for $\Lambda < 0$, so no horizons, and therefore a naked singularity appears at $r = 0$. For a genuine AdS_4 black hole solution we have to break the full supersymmetry, i.e. the mass has to be free to violate the BPS bound. If M is within a certain range, as explained in detail in e.g. [25], the solution has a proper horizon and describes a thermal AdS_4 black hole. There are some BPS generalizations of these solutions to the case of arbitrary number of vector multiplets [24], but the problem of naked singularities remains. For some further references on four-dimensional AdS black holes, including the non-extremal ones, see e.g. [26, 27].

Interestingly, recent developments in the AdS/CFT correspondence suggest that holographic superconductors are related to non-extremal static black holes in the presence of a charged scalar. Such cases will arise in $N = 2$ supergravity only when the hypermultiplets are gauged. Thus we will be able to give some statements about this interesting class of black holes, which we leave for section 3.2.2. In the rest of the paper we will mainly concentrate on the asymptotically flat BPS solutions with gauged hypers.

3 Black holes and spontaneous symmetry breaking

In this section we explain how to obtain a class of black hole solutions in gauged supergravity, starting from known solutions in ungauged supergravity. The main idea is simple: In gauged supergravity, one can give expectation values to some of the scalars (from both the vector and hypermultiplets) such that one breaks the gauge symmetry spontaneously in a maximally supersymmetric $N = 2$ vacua, specified by the conditions (2.23) or (2.24). Let us suppose for simplicity that the vacuum has zero cosmological constant, the argument can be repeated for $N = 2$ preserving anti-de Sitter vacua. Due to the Higgs mechanism some of the fields become massive, and as a consequence of the $N = 2$ preserving vacua, the gravitinos remain massless and the heavy modes form massive $N = 2$ vector multiplets. As a second step, we can set the heavy fields to zero, and the theory gets truncated to an ungauged $N = 2$ supergravity. These truncations are consistent due to the fact that supersymmetry is unbroken. Black hole solutions can then be found by taking any solution of the ungauged theory and augmenting it with the massive fields that were set to zero. In fact, it is clear from this procedure that one can even implement a non-BPS black hole solution of the ungauged theory into the gauged theory. It is also clear that this procedure works for non-abelian gaugings, as long as it is broken spontaneously to an abelian

subgroup with residual $N = 2$ supersymmetry. But for simplicity, and to streamline with subsequent sections, we will however only consider abelian gaugings. What is perhaps less clear, is to see if this procedure gives the most general black hole solutions. In other words, one can look for other solutions in which the massive scalars are non-trivial (i.e. with scalar hair). This is the subject of section 4.6.2, where we investigate the conditions for which new BPS black holes with scalar hair exist.

Let us now illustrate the above mechanism in some more detail. We restrict ourselves first to spontaneous symmetry breaking in Minkowski vacua, where one has $\langle P_\Lambda^x \rangle = 0$ and $\langle \tilde{k}_\Lambda^u L^\Lambda \rangle = 0$ according to (2.23). At such a point, the resulting potential is zero, see (2.16), as required by a Minkowski vacuum. After the hypermultiplet scalar fields take their vacuum expectation value, the Lagrangian (2.15) contains a mass-term for some of the gauge fields, given by

$$\mathcal{L}_{\text{mass}}^V = M_{\Lambda\Sigma} A_\mu^\Lambda A^{\mu\Sigma} , \quad M_{\Lambda\Sigma} \equiv g^2 \langle h_{uv} \tilde{k}_\Lambda^u \tilde{k}_\Sigma^v \rangle . \quad (3.1)$$

There is no contribution to the mass matrix for the vector fields coming from expectation values of the vector multiplet scalars, since the gauging was chosen to be abelian. The number of massive vectors is then given by the rank of $M_{\Lambda\Sigma}$, and as h_{uv} is positive definite, one has $\text{rank}(M_{\Lambda\Sigma}) = \text{rank}(\tilde{k}_\Lambda^u)$. Hence, the massive vector fields are encoded by the linear combinations $\tilde{k}_\Lambda^u A_\mu^\Lambda$. Similarly, some of the vector and hypermultiplet scalars acquire a mass, determined by expanding the scalar potential,

$$V = 4h_{uv} \tilde{k}_\Lambda^u \tilde{k}_\Sigma^v \bar{L}^\Lambda L^\Sigma + (g^{i\bar{j}} f_i^\Lambda \bar{f}_{\bar{j}}^\Sigma - 3\bar{L}^\Lambda L^\Sigma) P_\Lambda^x P_\Sigma^x , \quad (3.2)$$

to quadratic order in the fields. Then one reads off the mass matrix, and in general there can be off-diagonal mass terms between vector and hypermultiplet scalars. Massive vector multiplets can then be formed out of a massive vector, a massive complex scalar from the vector multiplet, and 3 hypermultiplet scalars. The fourth hypermultiplet scalar is the Goldstone mode that is eaten by the vector field. We will illustrate this more explicitly in some concrete examples below.

Upon setting the massive fields to zero (or integrating them out), one obtains a supergravity theory with only massless fields. Because of $\langle P_\Lambda^x \rangle = 0$, the mass matrix for the gravitinos is zero as follows from (2.21). Therefore, the resulting theory is an ungauged supergravity theory of the type discussed in section 2.1. Black hole solutions can then be simply copied from the results in section 2.2. By going through the Higgs mechanism in reverse order, one can uplift this solution easily to the gauged theory by augmenting it with the necessary expectation values of the scalars. It is then clear that the black hole solution is not charged with respect to the gauge fields that acquired a mass.

The situation for spontaneous symmetry breaking in an AdS vacua is similar. To generate a negative cosmological constant from the potential (2.16), we must have a $\langle P_\Lambda^x \rangle \neq 0$ in the vacuum. The conditions for unbroken $N = 2$ supersymmetry are given in (2.24). After expanding the fields around this vacuum, one can truncate the theory further to a Lagrangian with a bare cosmological constant, in which one can construct black hole solutions of the type discussed in section 2.4. We will discuss an example at the end of this section.

3.1 Solution generating technique

We now elaborate on constructing the black hole solutions more explicitly. As explained above, the general technique is to embed a (BPS) solution in ungauged supergravity into a gauged supergravity. The considerations in this subsection also apply for the more general case of non-abelian gaugings, although we are mainly interested here in the abelian case. First, to illustrate the systematics of our procedure, we analyze a simpler setup in which we embed solutions from pure supergravity into a model with vector multiplets only. Then we extend the models to include both hypermultiplets and vector multiplets, i.e. the most general (electrically) gauged supergravities. We always consider solutions with vanishing fermions, i.e. the discussion concerns only the bosonic fields.

3.1.1 Vector multiplets

We start from pure $N = 2$ supergravity, i.e. only the gravity multiplet normalized as $\mathcal{L} = \frac{1}{2}R(g) - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \Lambda$. Let us assume we have found a solution of this Lagrangian, which we denote by $\mathring{g}_{\mu\nu}, \mathring{F}_{\mu\nu}$. We can embed this into a supergravity theory with only vector multiplets as follows. If we have a theory with (gauged) vector multiplets we can find a corresponding solution to it by satisfying

$$\nabla_\mu z^i = 0, \quad G_{\mu\nu}^i = 0, \quad k_\Lambda^i \bar{L}^\Lambda = 0. \quad (3.3)$$

Note that the integrability condition following from $\nabla_\mu z^i = 0$ is always satisfied given the other constraints ⁶. We further have the relations

$$g_{\mu\nu} = \mathring{g}_{\mu\nu}, \quad \sqrt{2I_{\Lambda\Sigma}\bar{L}^\Lambda\bar{L}^\Sigma}T_{\mu\nu}^- = \mathring{F}_{\mu\nu}^-. \quad (3.4)$$

⁶Also note that we have used the Killing vectors k_Λ^i that specify a gauged isometry $\nabla_\mu z^i = \partial_\mu z^i + g k_\Lambda^i A_\mu^\Lambda$ on the vector multiplet scalar manifold. These automatically vanish if the isometry is abelian, and therefore will not be discussed further in this paper. The formulas here are still valid for any gauged isometry.

The last equality is to be used for determining $T_{\mu\nu}^-$. Then we can find the solution for our new set of gauge field strengths by $F_{\mu\nu}^{\Lambda-} = i\bar{L}^{\Lambda}T_{\mu\nu}^-$ since we already know that $G_{\mu\nu}^i = 0$.

The new configuration will, by construction, satisfy all equations of motion of the theory and will preserve the same amount of supersymmetry (if any) as the original one. This can be checked explicitly from the supersymmetry transformation rules (2.18) and (2.20) combined with the results from our previous paper [22]. Indeed (3.3) comes from imposing the vanishing of (2.18), while (3.4) is required by the Einstein equations. We will give a more explicit realization of this procedure in section 3.2.2.

3.1.2 Hypermultiplets

Given any solution of $N = 2$ supergravity with no hypermultiplets, we can obtain a new solution with (gauged) hypermultiplets preserving the same amount of supersymmetry as the original one. We require the theory to remain the same in the other sectors (vector and gravity multiplets with solution $\mathring{g}_{\mu\nu}, \mathring{F}_{\mu\nu}^{\Lambda}, \mathring{z}^i$) and impose some additional constraints that have to be satisfied in addition to the already given solution. We then simply require the fields of our new theory to be

$$g_{\mu\nu} = \mathring{g}_{\mu\nu} , \quad F_{\mu\nu}^{\Lambda} = \mathring{F}_{\mu\nu}^{\Lambda} , \quad z^i = \mathring{z}^i , \quad (3.5)$$

under the following restriction that has to be solved for the hypers. Here we are left with two cases: the original theory was either with or without Fayet-Iliopoulos (FI) terms (cosmological constant). In absence of FI terms, a new solution after adding hypers is given by imposing the constraints:

$$\nabla_{\mu}q^u = 0 \Rightarrow \tilde{k}_{\Lambda}^u F_{\mu\nu}^{\Lambda} = 0 , \quad P_{\Lambda}^x = 0 , \quad \tilde{k}_{\Lambda}^u L^{\Lambda} = 0 , \quad (3.6)$$

while in the case of original solution with FI terms we have a solution after adding hypers (thus no longer allowing for FI terms but keeping $P_{\Lambda}^x L^{\Lambda}$ the same) with:

$$\nabla_{\mu}q^u = 0 \Rightarrow \tilde{k}_{\Lambda}^u F_{\mu\nu}^{\Lambda} = 0 , \quad P_{\Lambda}^x f_i^{\Lambda} = 0 , \quad \epsilon^{xyz} P_{\Lambda}^y P_{\Sigma}^z L^{\Lambda} \bar{L}^{\Sigma} = 0 , \quad \tilde{k}_{\Lambda}^u L^{\Lambda} = 0 . \quad (3.7)$$

The new field configuration (given it can be found from the original data) again satisfies all equations of motion and preserves the same amount of supersymmetry as the original one. This is true because the susy variations of gluinos and gravitinos remain the same as in the original solution, and also the variations for the newly introduced hyperinos are zero.

3.1.3 Vector and hypermultiplets

This case is just combining the two cases above. If we start with no FI terms the new solution will be generated by imposing equations (3.6) and (3.3). If we have a solution with a cosmological constant we need to impose (3.7) and (3.3). Then the integrability condition following from $\nabla_\mu q^u = 0$ is automatically satisfied in both cases, using relations (3.4).

3.2 Examples

3.2.1 The STU model with gauged universal hypermultiplet

Here we discuss an example to illustrate explicitly the procedure outlined above. Let us consider an $N = 2$ theory with the universal hypermultiplet. Its quaternionic metric and isometries are given in C, and isometry 5 is chosen to be gauged. This allows for asymptotically flat black holes, since we can find solutions of (3.6), as we shall see below⁷. The quaternionic Killing vector and moment maps are given by

$$\tilde{k}_\Lambda = a_\Lambda (2R\partial_R + u\partial_u + v\partial_v + 2D\partial_D), \quad (3.8)$$

$$\vec{P}_\Lambda = a_\Lambda \left\{ -\frac{u}{\sqrt{R}}, \frac{v}{\sqrt{R}}, -\frac{D}{R} \right\}, \quad (3.9)$$

with a_Λ arbitrary constants.

In the vector multiplet sector we take the so-called STU model, based on the prepotential

$$F = \frac{X^1 X^2 X^3}{X^0}, \quad (3.10)$$

together with $z^i = \frac{X^i}{X^0}; i = 1, 2, 3$. The gauge group is $U(1)^3$, but it will be broken to $U(1)^2$ in the supersymmetric Minkowski vacua, in which we construct the black hole solution. The conditions for a fully BPS Minkowski vacuum require $F_{\mu\nu}^{vev} = 0, z^{i,vev} = \langle z^i \rangle = \langle b^i \rangle + i\langle v^i \rangle, u^{vev} = v^{vev} = D^{vev} = 0, R^{vev} = \langle R \rangle$, with arbitrary constants $\langle z^i \rangle$ and $\langle R \rangle$. Moreover, from (2.23), the vector multiplets scalar vevs must obey $a_\Lambda L^{\Lambda,vev} = 0$ (which is an equation for the $\langle z^i \rangle$'s). Then, after expanding around this vacuum, the mass terms for the scalar

⁷A suitable combination of isometries 1 and 4 would also do the job. Note that typically in string theory isometry 5 gets broken perturbatively while 1 and 4 remain also at quantum level. For the present discussion it is irrelevant which one we choose since we are not trying to directly obtain the model from string theory.

fields are given by the quadratic terms in (3.2). Now, if we make the definition $z \equiv a_\Lambda L^\Lambda$, we have $z^{vev} = 0$. Expanding the first term in (3.2) gives the mass term for z ,

$$\left(4h_{uv}\tilde{k}_\Lambda^u\tilde{k}_\Sigma^v\bar{L}^\Lambda L^\Sigma\right)^{quadratic} = 16z\bar{z}.$$

Expanding the second term to quadratic order gives the mass for three of the hypers:

$$(g^{i\bar{j}}f_i^\Lambda\bar{f}_{\bar{j}}^\Sigma P_\Lambda^x P_\Sigma^x)^{quadratic} = \frac{a_i^2\langle v^i \rangle^2}{\langle v^1 v^2 v^3 \rangle \langle R \rangle} \left(u^2 + v^2 + \frac{D^2}{\langle R \rangle}\right),$$

while the third term vanishes at quadratic order and does not contribute to the mass matrix of the scalars.

Therefore two of the six vector multiplet scalars become massive (i.e. the linear combination given by our definition for z), together with three of the hypers. The fourth hyper R remains massless and is eaten up by the massive gauge field $a_\Lambda A_\mu^\Lambda$ (with mass 4 given by 3.1). Thus we are left with an effective $N = 2$ supergravity theory of one massive and two massless vector multiplets and no hypermultiplets, which can be further consistently truncated to only include the massless modes. One can then search for BPS solutions in the remaining theory and the prescription for finding black holes is again the one given by Behrndt, Lüster and Sabra and explained in section 2.2.

We now construct the black hole solution more explicitly, following the solution generating technique of section (3.1). For this, we need to satisfy (3.5) and (3.6). The condition $P_\Lambda^x = 0$ fixes $u = v = D = 0$ and the remaining non-zero Killing vectors are $k_\Lambda^R = 2Ra_\Lambda$. Now we have to satisfy the remaining conditions $\tilde{k}_\Lambda^u X^\Lambda = 0$ and $\tilde{k}_\Lambda^u F_{\mu\nu}^\Lambda = 0$. To do so, we use the BLS solution of the STU model. For simplicity we take the static limit $\omega_m = 0$, discussed in detail in section 4.6 of [5]. The solution is fully expressed in terms of the harmonic functions

$$H_0 = h_0 + \frac{q_0}{r}, \quad \tilde{H}^i = h^i + \frac{p^i}{r}, \quad i = 1, 2, 3, \quad (3.11)$$

under the condition that one of them is negative definite. The sections then read

$$X^0 = \sqrt{-\frac{\tilde{H}^1 \tilde{H}^2 \tilde{H}^3}{4H_0}}, \quad X^i = -i\frac{\tilde{H}^i}{2}, \quad (3.12)$$

with metric function

$$e^{-\mathcal{K}} = \sqrt{-4H_0 \tilde{H}^1 \tilde{H}^2 \tilde{H}^3}. \quad (3.13)$$

In this case $F_{mn}^0 = 0$ and the F_{mn}^i components (here m, n are the spatial indices) are expressed solely in terms of derivatives of \tilde{H}^i . After evaluating the period matrix we

obtain $F_{mt}^i = 0$ and F_{mt}^0 are given in terms of derivatives of H_0, \tilde{H}^i . Thus the equations $\tilde{k}_\Lambda^R X^\Lambda = 0$ and $\tilde{k}_\Lambda^R F_{\mu\nu}^\Lambda = 0$ lead to

$$a_0 = 0, \quad a_i h^i = 0, \quad a_i p^i = 0. \quad (3.14)$$

The solution is qualitatively the same as the original one, but the charges p^i and the asymptotic constants h^i are now related by (3.14). So effectively, the number of independent scalars and vectors is decreased by one, consistent with the results from spontaneous symmetry breaking. The usual attractor mechanism for the remaining, massless vector multiplet scalars holds while for the hypermultiplet scalars we know that $u = v = D = 0$ and R is fixed to an arbitrary constant everywhere in spacetime with no boundary conditions at the horizon. In other words, the hypers are not “attracted”.

Our construction can be generalized for non-BPS solutions as well. In the particular case of the STU model, we can obtain a completely analogous, non-BPS, solution by following the procedure described in [28]. We flip the sign of one of the harmonic functions in (2.7) such that

$$e^{-\kappa} = \sqrt{4H_0\tilde{H}^1\tilde{H}^2\tilde{H}^3}. \quad (3.15)$$

This solution preserves no supersymmetry, but it is extremal. By following our procedure above, we can embed this solution into the gauged theory.

3.2.2 Asymptotically AdS black holes

Here we give a simple but yet qualitatively very general example of how to apply the procedure outlined above to find asymptotically anti-de Sitter black hole solutions with gauged hypers, starting from already known black hole solutions without hypers. In this case we start from a solution of pure supergravity and add abelian gauged vector multiplets and hypermultiplets. Alternatively, one can think of it as breaking the gauge symmetry such that all hyper- and vector multiplets become massive, and one is left with a gravity multiplet with cosmological constant. Here we already know the full classification of black hole solutions, as described in section 2.4.

An already worked out example in section 4.2 of [22] is the case of the gauged supergravity, arising from a consistent reduction to four dimensions of M-theory on a Sasaki-Einstein₇ manifold [29]. The resulting low-energy effective action has a single vector multiplet and a single hypermultiplet (the universal hypermultiplet). The special geometry prepotential is given by

$$F = \sqrt{X^0(X^1)^3},$$

with $X^\Lambda = \{1, \tau^2\}$, where τ is the vector multiplet scalar, and the isometries on the UHM are given by

$$\tilde{k}_0 = 24\partial_D - 4v\partial_u + 4u\partial_v, \quad \tilde{k}_1 = 24\partial_D, \quad (3.16)$$

which is combination of isometries 1 and 4 from appendix C. The corresponding moment maps are given by

$$P_0^1 = \frac{4v}{\sqrt{R}}, \quad P_0^2 = \frac{4u}{\sqrt{R}}, \quad P_0^3 = 4 - \frac{12 + 4(u^2 + v^2)}{R}, \quad (3.17)$$

$$P_1^1 = 0, \quad P_1^2 = 0, \quad P_1^3 = -\frac{12}{R}. \quad (3.18)$$

Maximally supersymmetric AdS_4 vacua were found in [22]. The condition (2.24) fixes the values of the vector multiplet scalar $\tau^{vev} \equiv (\tau_1 + i\tau_2)^{vev} = i$ and two of the four hypers $u^{vev} = v^{vev} = 0$. The third ungauged hyper, which is the dilaton, is fixed to the constant non-zero value $R^{vev} = 4$. The remaining hypermultiplet scalar is an arbitrary constant $D^{vev} = \langle D \rangle$. All the gauge fields have vanishing expectation values at this fully supersymmetric AdS_4 vacuum. If we now expand the scalar field potential (3.2) up to second order in fields we obtain the following mass terms

$$V^{quadratic} = -12 + 138(\tau_1^2 + \tau_2^2) + \frac{3}{4}R^2 + 6R\tau_2 + 10(u^2 + v^2). \quad (3.19)$$

We can see that three of the hyperscalars and the (complex) vector multiplet scalar acquire mass. There is also a mass term $m^2 = 36$ for the gauge field $A_0 + A_1$, this field thus eats up the remaining massless hyperscalar D . So we observe the formation of a massive $N = 2$ vector multiplet consisting of one massive vector and five massive scalars, and we can consistently set all these fields to zero. The resulting Lagrangian is that of pure $N = 2$ supergravity with a cosmological constant $\Lambda = -12$. Using the static class of black hole solutions of (2.26), it is straightforward to provide a solution of the gauged supergravity theory. All the solutions described in section 2.4 will also be solutions in our considered model as they obey the Einstein-Maxwell equations of pure supergravity.

4 1/2 BPS solutions

In this section we will take a more systematic approach to studying the supersymmetric solutions of (2.15). We search for a solution where the expectation values of the fermions are zero. This implies that the supersymmetry variations of the bosons should be zero. The vanishing of the supersymmetry variations (2.18)-(2.20) then guarantees some amount

of conserved supersymmetry. Depending on the number of independent components of the variation parameters ε_A we will have different amount of conserved supersymmetry. Here we will focus on particular solutions preserving (at least) 4 supercharges, i.e. half-BPS configurations. A BPS configuration has to further satisfy the equations of motion in order to be a real solution of the theory, so we also impose those. The fermionic equations of motion vanish automatically, so we are left with the equations of motion for the graviton $g_{\mu\nu}$, the vector fields A_μ^Λ , and the scalars z^i and q^u . We will come to the relation between the BPS constraints and the field equations in due course, but we first introduce some more relations for the Killing spinors ε_A .

4.1 Killing spinor identities

We will make use of the approach [30] where one first assumes the existence of a Killing spinor. From this spinor, various bilinears are defined, whose properties constrain the form of the solution to a degree where a full classification is possible. We use this method in $D = 4, N = 2$, which is generalizing the main results of [9] to include hypermultiplets in the description. As it later turns out, we cannot completely use this method to classify all the supersymmetric configurations, but the method nevertheless gives useful information.

We define ε_A to be a Killing spinor if it solves the gravitino variation $\delta_\varepsilon \psi_{\mu A} = 0$, defined in (2.20), and assume ε_A to be a Killing spinor in the remainder of this article. Such spinors anti-commute, but we can expand them on a basis of Grassmann variables and only work with the expansion coefficients. This leads to a commuting spinor, which we also denote with ε_A , and we define⁸

$$\begin{aligned}\bar{\varepsilon}_A &\equiv i(\varepsilon^A)^\dagger \gamma_0 , \\ X &\equiv \frac{1}{2} \epsilon^{AB} \bar{\varepsilon}_A \varepsilon_B , \\ V_\mu{}^A{}_B &\equiv i \bar{\varepsilon}^A \gamma_\mu \varepsilon_B , \\ \Phi_{AB\mu\nu} &\equiv \bar{\varepsilon}_A \gamma_{\mu\nu} \varepsilon_B .\end{aligned}\tag{4.1}$$

We now show that this implies that $V^\mu \equiv V_\mu{}^A{}_A$ is a Killing vector. For its derivatives we find

$$\begin{aligned}\nabla_\mu V_\nu{}^A{}_B &= i\delta^A{}_B (T_{\mu\nu}^+ X - T_{\mu\nu}^- \bar{X}) - g_{\mu\nu} (S^{AC} \epsilon_{CB} X - S_{BC} \epsilon^{AC} \bar{X}) \\ &\quad - i(\epsilon^{AC} T_\mu{}^{+\rho} \Phi_{CB\rho\nu} + \epsilon_{BC} T_\mu{}^{-\rho} \Phi^{AC}{}_{\nu\rho}) - (S^{AC} \Phi_{CB\mu\nu} + S_{BC} \Phi^{AC}{}_{\mu\nu}) .\end{aligned}\tag{4.2}$$

⁸We will be brief on some technical points of the discussion, and refer to [9] for more information.

The second and third term are traceless, so they vanish when we compute $\nabla_\mu V_\nu$. The other terms are antisymmetric in $\mu\nu$, so this proves

$$\nabla_\mu V_\nu + \nabla_\nu V_\mu = 0 , \quad (4.3)$$

thus V_μ is a Killing vector. We make the decomposition $V^A{}_{B\mu} = \frac{1}{2}V_\mu\delta^A{}_C + \frac{1}{\sqrt{2}}\sigma^{xA}{}_B V_\mu^x$ and using Fierz identities one finds

$$V_\mu{}^A{}_B V_\nu{}^B{}_A = V_\mu V_\nu - \frac{1}{2}g_{\mu\nu}V^2 . \quad (4.4)$$

One can show that $V_\mu V^\mu = 4|X|^2$, which shows that the Killing vector V_μ is timelike or null. For the remainder of this paper we restrict ourselves to a timelike Killing spinor ansatz, defined as one that leads to a timelike Killing vector. We make this choice, as our goal is to find stationary black hole solutions, which always have a timelike isometry⁹. In this case, by definition, $V_\mu V^\mu = 4|X|^2 \neq 0$, so we can solve (4.4) for the metric as

$$g_{\mu\nu} = \frac{1}{4|X|^2} (V_\mu V_\nu - 2V_\mu^x V_\nu^x) . \quad (4.5)$$

It follows that

$$V_\mu = g_{\mu\nu}V^\nu = V_\mu - \frac{1}{2|X|^2}V_\mu^x (V_\nu^x V^\nu) , \quad (4.6)$$

so $V_\mu^x V^\mu = 0$. We define a time coordinate by $V^\mu \partial_\mu = \sqrt{2}\partial_t$, which implies $V_t^x = 0$. We decompose $V_\mu dx^\mu = 2\sqrt{2}X\bar{X}(dt + \omega)$, where the factor in front of dt follows from $V^2 = 4X\bar{X}$ and ω has no dt component. The metric is then given by

$$ds^2 = 2|X|^2(dt + \omega)^2 - \frac{1}{2|X|^2}\gamma_{mn}dx^m dx^n , \quad (4.7)$$

where $|X|, \omega$ and γ_{mn} are independent of time.

Now we are ready to make a relation between the susy variations (2.18–2.20) and the equations of motion, using an elegant and simple argument of Kallosh and Ortin [31] that was later generalized in [9]. Assuming the existence of (any amount of) unbroken supersymmetry, one can derive a set of equations relating the equations of motion for the bosonic fields with derivatives of the bosonic susy variations. For our chosen theory these read:

$$\begin{aligned} \mathcal{E}_\Lambda^\mu i f_i^\Lambda \gamma_\mu \varepsilon^A \epsilon_{AB} + \mathcal{E}_i \varepsilon_B &= 0 , \\ \mathcal{E}_a^\mu (-i\gamma^a \varepsilon^A) + \mathcal{E}_\Lambda^\mu (2\bar{L}^\Lambda \varepsilon_B \epsilon^{AB}) &= 0 , \\ \mathcal{E}_u \mathcal{U}_{\alpha A}^u \varepsilon^A &= 0 , \end{aligned} \quad (4.8)$$

⁹We furthermore assume, or restrict to the cases, that the stationary BPS black hole has a time-like Killing vector which can be written as a bilinear in the Killing spinor.

where \mathcal{E} is the equation of motion for the corresponding field in subscript. More precisely, \mathcal{E}_a^μ is the equation for the vielbein e_μ^a (the Einstein equations), \mathcal{E}_Λ^μ corresponds to A_μ^Λ (the Maxwell equations), \mathcal{E}_u corresponds to q^u and \mathcal{E}_i to z^i . Now, let us assume that the Maxwell equations are satisfied, $\mathcal{E}_\Lambda^\mu = 0$. If we multiply from the left each of the remaining terms in the three equations by $\overline{\varepsilon^B}$ and $\overline{\varepsilon^B}\gamma^\nu$ and use the fact that the Killing spinor is timelike such that $X \neq 0$ we directly obtain that the remaining field equations are satisfied. So, apart from the BPS conditions, only the Maxwell equations

$$\epsilon^{\mu\nu\rho\sigma}\partial_\nu G_{\Lambda\rho\sigma} = -gh_{uv}\tilde{k}_\Lambda^u\nabla^\mu q^v, \quad (4.9)$$

need to be explicitly verified.

4.2 Killing spinor ansatz

Contracting the gaugino variation (2.18) with ε_A we find the condition

$$0 = -2i\bar{X}\nabla_\mu z^i + 4iG_{\rho\mu}^{-i}V^\rho - igk_\Lambda^i\bar{L}^\Lambda V_\mu - \sqrt{2}gg^{i\bar{j}}\bar{f}_{\bar{j}}^\Lambda P_\Lambda^x V_\mu^x. \quad (4.10)$$

Using this to eliminate $\nabla_\mu z^i$ and plugging back into $\delta\lambda^{iA} = 0$ we find¹⁰

$$G_{\rho\mu}^{i-}\gamma^\mu (2iV^\rho\varepsilon^A - \bar{X}\gamma^\rho\epsilon^{AB}\varepsilon_B) + gg^{i\bar{j}}\bar{f}_{\bar{j}}^\Lambda P_\Lambda^x \left(-\frac{1}{\sqrt{2}}V_\mu^x\gamma^\mu\varepsilon^A + i\bar{X}\sigma^{xAB}\varepsilon_B \right) = 0. \quad (4.11)$$

It is here that we find an important difference with the ungauged theories. In the latter case, $g = 0$, and the second term is absent. Then, assuming that the gauge fields $G_{\rho\mu}^{i-}$ are non-zero, one can rewrite equation (4.11) as

$$\varepsilon^A + ie^{-i\alpha}\gamma_0\epsilon^{AB}\varepsilon_B = 0, \quad (4.12)$$

where $e^{i\alpha} \equiv \frac{X}{|X|}$. One has thus *derived* the form of the Killing spinor, which is not an ansatz anymore.

In gauged supergravity, $g \neq 0$, so there are various ways to solve equation (4.11). One could, for instance, generalize (4.12) to

$$\varepsilon^A = b\gamma^0\epsilon^{AB}\varepsilon_B + a_m^x\gamma^m\sigma^{xAB}\varepsilon_B. \quad (4.13)$$

Plugging this back into (4.11), one obtains BPS conditions on the fields which one can then try to solve. While this is hard in general, it has been done in a specific case. Namely, the

¹⁰One could, as done in e.g. [9], eliminate the gauge fields $G_{\rho\mu}^{i-}$ to obtain an equivalent relation.

ansatz used for the AdS-RN black holes in minimally gauged supergravity (with a bare cosmological constant), as analyzed by Romans [15], fits into (4.13), but not in (4.12). In fact, we will see later that with (4.12) one cannot find AdS black holes.

In the remainder of this article, we will use (4.12) as a particular ansatz, hoping to find new BPS black hole solutions that are asymptotically flat. The reader should keep in mind that more general Killing spinors are possible, even for asymptotically flat black holes, and therefore our procedure will most likely not be the most general. The search for BPS black holes that asymptote to AdS_4 , and their Killing spinors, will be postponed for future research.

4.3 Metric and gauge field ansatz

We will further make the extra assumption that the solution for the spacetime metric, field strengths and scalars, is axisymmetric, i.e. there is a well-defined axis of rotation, such that $\omega = \omega_\varphi d\varphi$ lies along the angle of rotation (we choose to call it φ) in (4.7). For a stationary axisymmetric black hole solution the symmetries constrain the metric not to depend on t and φ . These symmetries also constrain the scalars and gauge field strengths to depend only on the remaining coordinates, which we choose to call r and θ . We further assume $F_{r\theta}^\Lambda = 0$, such that (after also using the gauge freedom) we can set $A_r^\Lambda = A_\theta^\Lambda = 0$ for all Λ .

4.4 Gaugino variation

Plugging the ansatz (4.12) into the gaugino variation $\delta\lambda^{iA} = 0$ gives

$$P_\Lambda^x f_i^\Lambda = 0 \ , \tag{4.14}$$

and

$$\left(e^{-i\alpha} \partial_\mu z^i \gamma^\mu \gamma^0 + G_{\mu\nu}^{-i} \gamma^{\mu\nu} \right) \varepsilon_A = 0 \ . \tag{4.15}$$

The latter condition can be simplified further, but we will see in what follows that it automatically becomes simpler or gets satisfied in certain cases, so we will come back to (4.15) later. We will make use of condition (4.14) when solving the gravitino integrability conditions.

4.5 Hyperino variation

With the ansatz (4.12), setting the hyperino variation to zero gives the condition

$$e^{-i\alpha} \nabla_\mu q^u \gamma^\mu \gamma_0 + 2g \tilde{k}_\Lambda^u \bar{L}^\Lambda = 0 . \quad (4.16)$$

Using the independence of the gamma matrices, one finds

$$\begin{aligned} \nabla_r q^u &= \nabla_\theta q^u = 0 , \\ \nabla_\varphi q^u &= \omega_\varphi \nabla_t q^u , \\ \nabla_t q^u &= -\sqrt{2} g \tilde{k}_\Lambda^u (X \bar{L}^\Lambda + \bar{X} L^\Lambda) , \\ 0 &= \tilde{k}_\Lambda^u (\bar{X} L^\Lambda - X \bar{L}^\Lambda) . \end{aligned} \quad (4.17)$$

Using axial symmetry and the gauge choice for the vector fields, $A_r^\Lambda = A_\theta^\Lambda = 0$, it follows that $\nabla_r q^u = \partial_r q^u$ and $\nabla_\theta q^u = \partial_\theta q^u$, and these both vanish from the BPS conditions. Furthermore, the hypers cannot depend on t and φ , because this would induce such dependence also on the vector fields and complex scalars via the Maxwell equations (4.9). Thus the hypers cannot depend on any of the space-time coordinates, so they are constant. This will be important when we analyze the gravitino variation.

4.6 Gravitino variation

The gravitino equation reads

$$\nabla_\mu \varepsilon_A = -e^{-i\alpha} (T_{\mu\rho}^- \gamma^\rho \delta_A^C + g S_{AB} \epsilon^{BC} \gamma_\mu) \gamma_0 \varepsilon_C . \quad (4.18)$$

We study the integrability condition which follows from this equation. The explicit computation is presented in appendix B. The main result that we will first focus on is equation (B.9),

$$T_{\mu\nu}^- P_\Lambda^x L^\Lambda = 0 , \quad (4.19)$$

so that there are two separate cases: $T_{\mu\nu}^- = 0$ or $P_\Lambda^x L^\Lambda = 0$. We will study these two cases in different subsections.

4.6.1 $T_{\mu\nu}^- = 0$

In this case the integrability conditions imply that the space-time is maximally symmetric with constant scalar curvature $P_\Lambda^x L^\Lambda$, as further explained in appendix B.1 . This corresponds either to Minkowski space when $P_\Lambda^x L^\Lambda = 0$, or AdS_4 when the scalar curvature is

non-zero. Although there might be interesting half BPS solutions here, they will certainly not describe black holes.

4.6.2 $P_\Lambda^x = 0$

The second case is $P_\Lambda^x L^\Lambda = 0$. We combine this identity with $P_\Lambda^x f_i^\Lambda = 0$ from (4.14). We now obtain

$$P_\Lambda^x \begin{pmatrix} \bar{L}^\Lambda \\ f_i^\Lambda \end{pmatrix} = 0. \quad (4.20)$$

The matrix between brackets on the left hand side is invertible. This follows from the properties of special geometry, and we used it also in the characterization of the maximally supersymmetric vacua in [22]. We therefore conclude that $P_\Lambda^x = 0$. Next, we show that in this case we have enough information to solve the gravitino variation and give the metric functions.

From the definition (B.1) for $\nabla_\mu \varepsilon_A$, the quaternionic $Sp(1)$ connection $\omega_{\mu A}{}^B$ vanishes, as the hypers are constant by the arguments in section 4.5. Combining this with $P_\Lambda^x = 0$, we see that the gravitino variation (2.20) is precisely the same as in a theory without hypermultiplets and vanishing FI-terms. Thus our problem reduces to finding the most general solution of the gravitino variation in the ungauged theory. The answer, as proven by [9], is that this is the well-known BLS solution [5] for stationary black holes (or naked singularities and monopoles in certain cases). Thus we can use the BLS solution, which in fact also solves the gaugino variation (4.15). We now only have to impose the Maxwell equations, which are not the same as in the BLS setup, due to the gauging of the hypermultiplets.

The sections are again described by functions H_Λ and \tilde{H}^Λ , as in (2.7), although not all of them are harmonic. The metric and field strengths are given by (2.8), (2.13) and (2.14). In terms of our original description (4.7), we have that γ_{mn} is three-dimensional flat space and

$$e^\mathcal{K} = 2|X|^2. \quad (4.21)$$

In the ungauged case the Maxwell equations have no source term and the field strengths are thus described by harmonic functions, while now in our case they will be more complicated. We can then directly compare to the original BLS solution described in section 2.2 and see how the new equations of motion change it. At this point we have chosen the phase α in (4.12) to vanish, just as it does in the BLS solution. We can do this without any

loss of generality since an arbitrary phase just appears in the intermediate results for the symplectic sections (2.7), but drops out of the physical quantities such as the metric and the field strengths.

We repeat that the Maxwell equations are given by (4.9),

$$\epsilon^{\mu\nu\rho\sigma}\partial_\nu G_{\Lambda\rho\sigma} = -gh_{uv}\tilde{k}_\Lambda^u\nabla^\mu q^v, \quad (4.22)$$

with $G_{\mu\nu}$ defined as in (2.11). Since our Bianchi identities are unmodified, and the same as in BLS, we again solve them by taking the \tilde{H}^Λ 's to be harmonic functions. The difference is in the Maxwell equations.

We plug in the identities from (4.17), (2.8) and (2.14). The components of (4.22) with $\mu \neq t$ are then automatically satisfied. The only non-trivial equation follows from $\mu = t$, and reads

$$\square H_\Lambda = -2g^2 e^{-\kappa} h_{uv} \tilde{k}_\Lambda^u \tilde{k}_\Sigma^v X^\Sigma, \quad (4.23)$$

where \square is again the three-dimensional Laplacian in flat space. The left hand side is real, and so is the right hand side, as a consequence of the last equation in (4.17) and the fact that we have chosen the phase in $X/|X|$ (see (4.12) to vanish. In other words, X is real, and therefore also $\tilde{k}_\Lambda^u X^\Lambda$ is real.

We furthermore have a consistency condition for the field strengths. The gauge potentials appear in (4.17), but also in (2.14), and these should lead to the same solution. These consistency conditions were not present in the ungauged case, since in that case there are no restrictions on F^Λ from the hyperino variation. The constraints can be easily derived from the integrability conditions of (4.17), and are given by

$$\begin{aligned} \tilde{k}_\Lambda^u \tilde{H}^\Lambda &= 0, \\ \tilde{k}_\Lambda^u F_{r\varphi}^\Lambda &= -\tilde{k}_\Lambda^u \partial_r (\omega_\varphi e^\kappa X^\Lambda), \\ \tilde{k}_\Lambda^u F_{\theta\varphi}^\Lambda &= -\tilde{k}_\Lambda^u \partial_\theta (\omega_\varphi e^\kappa X^\Lambda), \\ \tilde{k}_\Lambda^u F_{rt}^\Lambda &= -\tilde{k}_\Lambda^u \partial_r (e^\kappa X^\Lambda), \\ \tilde{k}_\Lambda^u F_{\theta t}^\Lambda &= -\tilde{k}_\Lambda^u \partial_\theta (e^\kappa X^\Lambda). \end{aligned} \quad (4.24)$$

The first condition can always be satisfied as it merely implies that some of the harmonic functions \tilde{H}^Λ depend on the others (remember that the hypermultiplet scalars are constant, and therefore also the Killing vectors \tilde{k}_Λ^u). In more physical terms, this constraint decreases the number of magnetic charges by the rank of \tilde{k}_Λ^u . The other constraints have to be

checked against the explicit form of the field strengths (2.13) and (2.14). This cannot be done generically and has to be checked once an explicit model is taken.

In section 3, we explained how the vanishing of $\tilde{k}_\Lambda^u L^\Lambda$ and $\tilde{k}_\Lambda^u A_\mu$ led to a BPS solution using spontaneous symmetry breaking. We can see that also from the equations of this section. When $\tilde{k}_\Lambda^u L^\Lambda = 0$, the right hand side of (4.23) is zero. This equation is then solved by harmonic functions H_Λ . Furthermore, as \tilde{k}_Λ^u is constant, we can move it inside the derivatives in (4.24), so the right hand sides are zero. The left hand sides are zero as well, as $\tilde{k}_\Lambda^u F_{\mu\nu}^\Lambda = 0$. Finally, the condition $\tilde{k}_\Lambda^u \tilde{H}^\Lambda = 0$ is satisfied as $\tilde{k}_\Lambda^u L^\Lambda$ is already real.

5 Solutions with scalar hair

In this section, we search for solutions of the above BPS conditions that do not fall in the class described in section 3. They describe asymptotically flat black holes and would have non-trivial profiles for the massive vector and scalar fields, i.e. they would be distinguishable by the scalar hair degrees of freedom outside the black hole horizon. Remarkably, we could not find models with pure scalar hair solutions without the need to introduce some extra features, such as ghost modes or non-vanishing fermions. Below, we describe two examples of solutions that lead to at least one negative eigenvalue of the Kähler metric. We show that if we require strictly positive definite kinetic terms in the considered models, one cannot find scalar hair solutions, but only the ones described in section 3. It is of course hard to justify these ghost solutions physically. However, there have been cases in literature where this is not necessarily a problem, e.g. in Seiberg-Witten theory [32, 33] one has to perform duality transformation such that the kinetic terms remain positive definite. Whether a similar story holds in our case remains to be seen. If such duality transformations exist they will have to map the ghost black hole solutions of our abelian electrically gauged supergravity to proper black hole solutions, possibly of magnetically gauged supergravity. However, we cannot present any direct evidence for such a possibility.

5.1 Ghost solutions

Before we present our examples, we start with a general comment. We can obtain some more information from the Einstein equations. The trace of the Einstein equations reads

$$R = T^q + T^z + 4V , \tag{5.1}$$

where R is the Ricci scalar, and we have defined

$$T^q = -2h_{uv}\nabla_\mu q^u\nabla^\mu q^v, \quad T^z = -2g_{i\bar{j}}\partial_\mu z^i\partial^\mu \bar{z}^{\bar{j}}. \quad (5.2)$$

Using the BPS conditions in (4.17), one quickly finds $T^q = -2V$. Furthermore, as $\partial_t z^i = 0$, we find¹¹ $T^z \geq 0$, and $V \geq 0$ by equations (2.16) and the condition $P_\Lambda^x = 0$. We therefore find

$$R = T^z + 2V \geq 0, \quad (5.3)$$

as long as the metric $g_{i\bar{j}}$ is positive definite. So the BPS conditions forbid the Ricci scalar R to become negative. In our examples below, the metric components will show some oscillatory behavior, as a consequence of the non-linear differential equation (4.23). Therefore, their derivatives, and hence the Ricci scalar, will oscillate between positive and negative values. This would contradict the positivity bound (5.3), unless the Kähler metric $g_{i\bar{j}}$ contains regions in which it is not positive definite. We now discuss this in detail with two examples.

5.1.1 Quadratic prepotential

We start with two simple models, which have only one vector multiplet. They are described by the two prepotentials

$$F = -\frac{i}{2} (X^0 X^0 \pm X^1 X^1). \quad (5.4)$$

These lead to the special Kähler metrics

$$g_{z\bar{z}} = \frac{\mp 1}{(1 \pm z\bar{z})^2}, \quad (5.5)$$

where $z \equiv X^1/X^0$. With the upper sign, we therefore get a negative definite Kähler metric and the vector multiplet scalar is a ghost field. With the lower sign, we obtain a positive definite metric. We couple this to the universal hypermultiplet, and gauge isometry 5 from appendix C, using A_μ^1 as the gauge field. The condition $P_\Lambda^x = 0$ fixes $u = v = D = 0$ and the only non-vanishing component of the Killing vectors is then $\tilde{k}_1^R = 2Ra_1$, where a_1 is a constant.

From the relations (2.7) it follows that $X^0 = \frac{1}{2}(H_0 - i\tilde{H}^0)$ and $X^1 = \frac{1}{2}(\pm H_1 - i\tilde{H}^1)$. The Kähler potential (2.9) is then

$$e^{-\mathcal{K}} = 2 (X^0 \bar{X}^0 \pm X^1 \bar{X}^1). \quad (5.6)$$

¹¹Recall that our spacetime signature convention is $(+, -, -, -)$.

As we do not use A_μ^0 for the gauging, X^0 remains harmonic, such that even if the solution for X^1 is considerably different, we still have hope of producing a black hole by having X^1 as a small perturbation of the leading term X^0 in the metric function $e^{-\mathcal{K}}$. For simplicity, we restrict ourself to the spherically symmetric single-centered case, so now our constraints (4.24) lead to $\tilde{H}^1 = 0$ and $\tilde{k}_\Lambda^u F_{rt}^\Lambda = -\tilde{k}_\Lambda^u \partial_r (e^{\mathcal{K}} X^\Lambda)$. The latter eventually implies that \tilde{H}^0 is constant. Since we can absorb this constant by rescaling H_0 , we will set $\tilde{H}^0 = 0$. Thus we are left with $2X^0 = H_0 = \sqrt{2} + \frac{q_0}{r}$ ($q_0 > 0$), where we set the constant of the harmonic function to $\sqrt{2}$ to obtain canonically normalized Minkowski space as $r \rightarrow \infty$.

The metric is given by (2.8), where

$$e^{-\mathcal{K}} = \frac{1}{2} \left(\left(\sqrt{2} + \frac{q_0}{r} \right)^2 \pm H_1^2 \right). \quad (5.7)$$

The only undetermined function is H_1 , which is subject to the only equation left to be satisfied, (4.23), which in this case is given by

$$\square H_1 = \mp e^{-\mathcal{K}} H_1 = \mp \frac{1}{2} \left(\left(\sqrt{2} + \frac{q_0}{r} \right)^2 \pm H_1^2 \right) H_1, \quad (5.8)$$

after setting $g|k| = 1$. Besides the trivial solution $H_1 = 0$ (belonging to the class solutions from section 3), we could not find an analytic solution to these equations. We can analyze the differential equation as $r \rightarrow 0$ and $r \rightarrow \infty$. As $r \rightarrow \infty$, we require $e^{-\mathcal{K}} \rightarrow 1$, to obtain flat space at infinity¹². Likewise, we require, as $r \rightarrow 0$, that $e^{-\mathcal{K}} \rightarrow q^2 r^{-2}$, to obtain $AdS_2 \times S^2$ at the horizon. The constant q (which is not necessarily equal to q_0) determines the (equal) radii of AdS_2 and S^2 . If we solve (5.8) for large values of r , we have to solve $\square H_1 = \mp H_1$; for small values of r we have to solve $\square H_1 = \mp \frac{1}{2} q^2 r^{-2} H_1$.

- With the upper sign (the ghost model), we find the general solution

$$H_1 = A \frac{\cos(r)}{r} + B \frac{\sin(r)}{r}, \quad r \rightarrow \infty, \quad (5.9)$$

$$H_1 = C r^{-\frac{1}{2}-\frac{1}{2}} \sqrt{1-4q^2} + D r^{-\frac{1}{2}+\frac{1}{2}} \sqrt{1-4q^2}, \quad r \rightarrow 0. \quad (5.10)$$

As long as $4q^2 < 1$, all the asymptotics are fine.

- With the lower sign (the non-ghost model), we find the general solution

$$H_1 = A \frac{e^{-r}}{r} + B \frac{e^r}{r}, \quad r \rightarrow \infty, \quad (5.11)$$

¹²Perhaps one can relax this requirement, and generalize this analysis to include BPS domain walls, which have different boundary conditions. For a discussion in four dimensions, see e.g. [38].

$$H_1 = Cr^{-\frac{1}{2}-\frac{1}{2}}\sqrt{1+4q^2} + Dr^{-\frac{1}{2}+\frac{1}{2}}\sqrt{1+4q^2}, \quad r \rightarrow 0. \quad (5.12)$$

When B is nonzero, this violates the boundary condition that $e^{-\kappa} \rightarrow 1$ as $r \rightarrow \infty$, so we have to set $B = 0$. Likewise, we have to set $C = 0$. We will now prove that imposing such boundary conditions implies $H_1 = 0$. To do this, we use the identity

$$\int_0^\infty (rH_1)\partial_r^2(rH_1) dr = - \int_0^\infty \partial_r(rH_1)\partial_r(rH_1) dr + (rH_1)\partial_r(rH_1) \Big|_{r=0}^{r=\infty}. \quad (5.13)$$

Using (5.11) and (5.12) one finds that, for $B = C = 0$, the boundary term vanishes. On the left-hand side, we use (5.8), and we obtain (using $\square H_1 = r^{-1}\partial_r^2(rH_1)$)

$$\int_0^\infty H_1 e^{-\kappa} H_1 dr = - \int_0^\infty \partial_r(rH_1)\partial_r(rH_1) dr. \quad (5.14)$$

The left-hand side is non-negative, whereas the right-hand side is non-positive, so this proves $H_1 = 0$. This argument can easily be repeated for solutions with only axial symmetry.

We can plot the solution with the upper sign numerically with generic starting conditions, and the result is shown on Fig.1(a). The metric function gets oscillatory perturbations, while having its endpoints fixed to the desired values as shown on Fig.1(b).

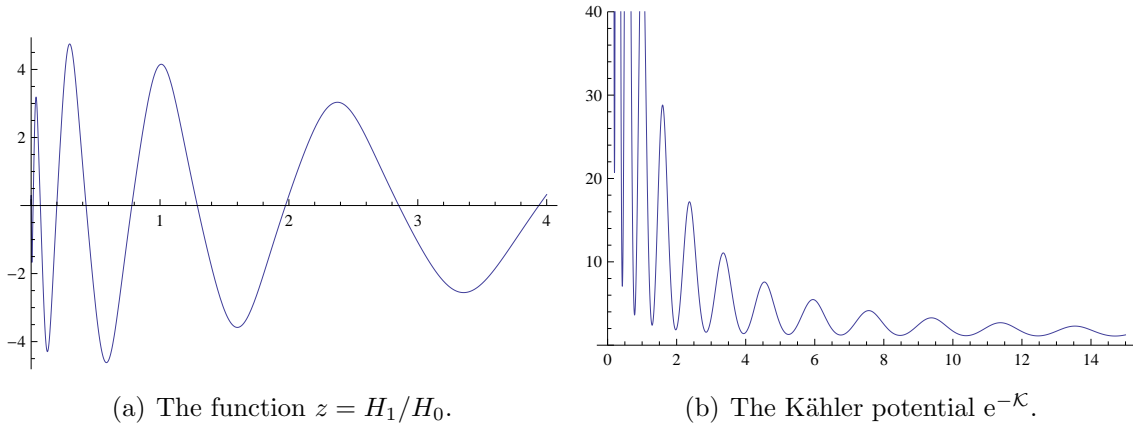


Figure 1: Plots of the solution to the differential equation (5.8) for $q_0 = 1$, using boundary conditions $H_1(1) = 10$ and $H'_1(1) = 1$. The scalar z approaches zero at the horizon at $r = 0$, and the Kähler potential $e^{-\kappa}$ approaches 1 as $r \rightarrow \infty$.

The function H_1 approaches zero as $r \rightarrow \infty$ in an oscillatory fashion, which can be seen in Fig.1(a). To investigate the behavior near the horizon at $r = 0$, we also checked that rH_1 approaches zero, and hence H_1 diverges slower than $1/r$. Both are in agreement with the asymptotic analysis above.

The numerics further show that the metric function for negative values of r yields the expected singularity at $r = -\frac{q_0}{\sqrt{2}}$. We conclude that this is indeed a black hole space-time, having one electric charge q_0 , and the fluctuations around the usual form of the metric are due to the effect of the abelian gauging of the hypermultiplet.

Let us now try to give a bit more physical interpretation of this new black hole spacetime. After more careful inspection of the solution, we see that at the horizon and asymptotically at infinity we again have supersymmetry enhancement, since the vector multiplet scalars are fixed to a constant value. It is interesting that the electric charge, associated to the broken gauge symmetry vanishes at the horizon, i.e. the black hole itself is not charged with q_1 exactly as in the normal case without ghosts. Yet there is a non-zero charge density for this charge everywhere in the spacetime outside the black hole, which is the qualitatively new feature of the ghost solutions. Clearly the fact that there is non-vanishing charge density everywhere in space-time does not change the asymptotic behavior, but it seems that it is physically responsible for the ripples that can be observed in the metric function on Fig. 1(b) (of course this is all related to the fact that we have propagating ghost fields). We should note that these are not the first rippled black hole solutions, similar behavior is found in the higher derivative ungauged solutions, e.g. in [34], where also one finds ghost modes in the resulting theory. The detailed analysis in section 4 of [34] holds in our case, i.e. the main physical feature of the ripples is that gravitational force changes from attractive to repulsive in some space-time points.

5.1.2 Cubic prepotential

The example above shows already the general qualitatively new features of this class of black holes with ghost fields, but is still not interesting from a string theory point of view, since Calabi-Yau compactifications lead to cubic prepotentials of the form

$$F = -\frac{\kappa_{ijk}X^iX^jX^k}{6X^0} . \quad (5.15)$$

The simplest case one can consider is the STU model of section 3.2.1. We coupled it to the universal hypermultiplet with a single gauged isometry and found it impossible to produce any new solutions. However, other choices of κ_{ijk} allow for interesting numerical solutions of (4.23). For this purpose we consider a relatively simple model with three vector multiplets:

$$F = \frac{(X^1)^3 - (X^1)^2X^2 - X^1(X^3)^2}{2X^0} . \quad (5.16)$$

We again use the universal hypermultiplet and gauge the same isometry as before, but we now use only A_μ^3 for our gauging. Again, the condition $P_\Lambda^x = 0$ fixes $u = v = D = 0$, and the only non-vanishing component of the Killing vector is $\tilde{k}_3^R = 2Ra_3$. In parts of moduli space this model exhibits proper Calabi-Yau behavior, i.e. the Kähler metric is positive definite, but there are regions where $g_{i\bar{j}}$ has negative eigenvalues (or $e^{-\kappa}$ becomes negative). There is no general expression for this so-called positivity domain; one has to analyze an explicit model to find the conditions.

For simplicity, we set $\tilde{H}^i = H_0 = 0$, so the non-vanishing functions are H_i and \tilde{H}^0 . Inverting (2.7) we obtain for the Kähler potential

$$e^{-\kappa} = \sqrt{2H_2} \sqrt{\tilde{H}^0} \left(H_1 + H_2 + \frac{H_3^2}{4H_2} \right). \quad (5.17)$$

We see that, as is commonly encountered in these models, one has to choose the signs of the functions H_i and \tilde{H}^0 such that this gives a real and positive quantity. With these we satisfy all conditions in (4.24) and are left to solve (4.23) that explicitly reads:

$$\square H_3 = -a_3^2 \tilde{H}^0 \left(H_1 + H_2 + \frac{H_3^2}{4H_2} \right) H_3, \quad (5.18)$$

where \tilde{H}^0 , H_1 and H_2 are harmonic functions, and we have set $g[\tilde{k}] = 1$ for convenience.

We impose the same boundary conditions, so as $r \rightarrow \infty$, we require $e^{-\kappa} \rightarrow 1$, to obtain flat space at infinity. Likewise, we require, as $r \rightarrow 0$, that $e^{-\kappa} \rightarrow q^2 r^{-2}$, to obtain $AdS_2 \times S^2$ at the horizon. Using (5.17), we then find that we have to solve

$$\begin{aligned} \square H_3 &= -a_3^2 q^2 r^{-2} H_3, & \text{as } r \rightarrow 0, \\ \square H_3 &= -a_3^2 c^2 H_3, & \text{as } r \rightarrow \infty, \end{aligned}$$

where c^2 is also a constant, specified by the asymptotics of \tilde{H}^0 , H_1 and H_2 . We therefore again find

$$H_3 = A \frac{\cos(a_3 c r)}{r} + B \frac{\sin(a_3 c r)}{r}, \quad \text{as } r \rightarrow \infty. \quad (5.19)$$

These functions are oscillating; therefore the Kähler potential (5.17) will also oscillate. This causes the Ricci scalar to become negative, which is in violation of the bound (5.3). Therefore, there is always a negative eigenvalue of the metric, corresponding to a ghost mode.

We could only find a numerical solution to this equation, and the results are qualitatively the same as the ones on figure 5.1.1, so we will omit them for this model.

It is therefore possible to find black hole solutions in these Calabi-Yau models, but they do contain regions in which scalars become ghost-like.

5.2 Fermionic hair

There is a different way of generating scalar hair with properly normalized positive-definite kinetic terms. As such, we can thereby avoid the ghost-like behavior of the previously discussed examples. The idea is simple and works for any solution that breaks some supersymmetry. By acting with the broken susy generators on a bosonic solution, we will turn on the fermionic fields to yield the fermionic zero modes. These fermionic zero modes solve the linearized equations of motion and produce fermionic hair. In turn, the fermionic hair sources the equations of motion for the bosonic field, and in particular, the scalar field equations will have a source term which is bilinear in the fermions. The solution of this equation produces scalar hair and can be found explicitly by iterating again with the broken supersymmetries. This iteration procedure stops after a finite number of steps and produces a new solution to the full non-linear equations of motion. By starting with a BPS black hole solution of the type discussed in section 3, one therefore produces new solutions with both fermionic and scalar hair. For a discussion on this for black holes in ungauged supergravity, see [35].

The explicit realization of this idea is fairly complicated since it requires to explicitly find the Killing spinors preserving supersymmetry. This can sometimes be done also just by considering the possible bosonic and fermionic deformations of the theory, as done in e.g. [36, 37] for black holes in ungauged supergravity. The extension of this hair-analysis to gauged supergravities would certainly be an interesting extension of our work.

6 Outlook

In this paper, we initiated the study of BPS black holes in $N = 2, D = 4$ gauged supergravities. An interesting class of solutions can be found through spontaneous symmetry breaking. They can be constructed explicitly by embedding known solutions of ungauged supergravity into the gauged theory. We also investigated the possibility of more general BPS black hole solutions, with scalar hair. Remarkably, we could not find static solutions without ripples in the spacetime geometry and ghost-like behavior for some of the scalar fields. It would be interesting to understand this better, prove a no-go theorem or see if there are ways to circumvent the ghost-problem, e.g. along the lines of section 5.2.

The BPS black hole solutions we considered in the second half of the paper were, as a consequence of the Killing spinor ansatz (4.12), asymptotically flat. To find solutions

which asymptote to anti-de Sitter space, one needs to generalize the Killing spinor ansatz to for instance, (4.13). Perhaps the coupling to the hypermultiplets then allows for BPS black hole solutions in AdS_4 which do not contain any naked singularities.

Finally, one would like to go beyond the two-derivative approximation and study the effects of higher order curvature terms in gauged supergravities. This is interesting since the thermodynamics of the black hole, in particular the Bekenstein-Hawking law, will change. As a consequence, the microscopic interpretation within flux compactifications of string theory, might also reveal new interesting phenomena for black hole physics and quantum gravity in general.

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A Conventions

We mainly follow the notation and conventions from [13], so we use a $\{+, -, -, -\}$ signature for the spacetime metric. Self-dual and anti-self-dual tensors are defined as

$$F_{\mu\nu}^{\pm} = \frac{1}{2} \left(F_{\mu\nu} \pm \frac{i}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \right), \quad (\text{A.1})$$

where $\epsilon_{0123} = 1$, and $F_{\mu\nu} \equiv \frac{1}{2}(\partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu})$ for abelian gauge fields.

Our gamma matrices satisfy

$$\begin{aligned} \{\gamma_a, \gamma_b\} &= 2\eta_{ab}, \\ [\gamma_a, \gamma_b] &\equiv 2\gamma_{ab}, \\ \gamma_5 &\equiv -i\gamma_0\gamma_1\gamma_2\gamma_3 = i\gamma^0\gamma^1\gamma^2\gamma^3. \end{aligned} \quad (\text{A.2})$$

In addition, they can be chosen such that

$$\gamma_0^{\dagger} = \gamma_0, \quad \gamma_0\gamma_i^{\dagger}\gamma_0 = \gamma_i, \quad \gamma_5^{\dagger} = \gamma_5, \quad \gamma_{\mu}^* = -\gamma_{\mu}, \quad (\text{A.3})$$

and an explicit example of such a basis is the Majorana basis, given by

$$\begin{aligned} \gamma^0 &= \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, & \gamma^1 &= \begin{pmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix}, & \gamma^2 &= \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, \\ \gamma^3 &= \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & -i\sigma^1 \end{pmatrix}, & \gamma_5 &= \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}, \end{aligned} \quad (\text{A.4})$$

where the σ^i are the Pauli matrices.

B Integrability conditions

The supercovariant derivative in (2.20) is defined as

$$\nabla_{\mu}\varepsilon_A = \left(\partial_{\mu} - \frac{1}{4}\omega_{\mu}^{ab}\gamma_{ab} \right) \varepsilon_A + \frac{i}{2}A_{\mu}\varepsilon_A + \omega_{\mu A}^B \varepsilon_B. \quad (\text{B.1})$$

The connections A_{μ} and $\omega_{\mu A}^B$ are associated to the special Kähler and quaternion-Kähler manifolds, respectively; we refer to [13] for more details. The curvature computed from these expressions is, in a theory with neutral vector multiplet scalars, given by [22]

$$\begin{aligned} [\nabla_{\mu}, \nabla_{\nu}]\varepsilon_A &= -\frac{1}{4}R_{\mu\nu}^{ab}\gamma_{ab}\varepsilon_A - g_{i\bar{j}}\partial_{[\mu}z^i\partial_{\nu]}z^{\bar{j}}\varepsilon_A \\ &\quad + 2i\Omega_{uvA}^B\nabla_{[\mu}q^u\nabla_{\nu]}q^v\varepsilon_B + ig\sigma^x{}_A{}^B F_{\mu\nu}^{\Lambda}P_{\Lambda}^x\varepsilon_B. \end{aligned} \quad (\text{B.2})$$

If ε_A is a Killing spinor, it obeys

$$\nabla_\mu \varepsilon_A = -\epsilon_{AB} T_{\mu\rho}^- \gamma^\rho \varepsilon^B - ig S_{AB} \gamma_\mu \varepsilon^B, \quad (\text{B.3})$$

hence the commutator is

$$\begin{aligned} [\nabla_\mu, \nabla_\nu] \varepsilon_A = & -\epsilon_{AB} D_\mu T_{\nu\rho}^- \gamma^\rho \varepsilon^B + \frac{g}{2} \sigma_{AB}^x \nabla_\mu P^x \gamma_\nu \varepsilon^B - (\mu\nu) \\ & + T_{\nu\rho}^- \gamma^\rho T_{\mu\sigma}^+ \gamma^\sigma \varepsilon_A - (\mu\nu) \\ & - \frac{g}{2} T_{\nu\rho}^- \gamma^\rho \gamma_\mu P_\Lambda^x \bar{L}^\Lambda \sigma^x{}_A{}^C \varepsilon_C + \frac{g}{2} T_{\mu\rho}^+ \gamma_\nu \gamma^\rho P_\Lambda^x L^\Lambda \sigma^x{}_A{}^C \varepsilon_C - (\mu\nu) \\ & + \frac{g^2}{2} (\delta_A{}^C P^x \bar{P}^x - i \epsilon^{xyz} \sigma^x{}_A{}^C P^y \bar{P}^z) \gamma_{\mu\nu} \varepsilon_C. \end{aligned} \quad (\text{B.4})$$

One can now equate (B.4) to (B.2). We use (4.12) to eliminate ε^A in terms of ε_A and for convenience define $b \equiv -ie^{i\alpha}$ and $P^x \equiv P_\Lambda^x L^\Lambda$. The remaining equation should hold for any choice of ε_A . We can then use the independence of the gamma matrices and the SU(2) matrices $\epsilon_{AB}, \sigma_{AB}^x$ to find the following list of conditions:

1. Terms proportional to ϵ_{AB} , with no gamma matrices,

$$b D_\mu T_{\nu 0}^- - (\mu\nu) = -g_{i\bar{j}} \partial_{[\mu} z^i \partial_{\nu]} z^{\bar{j}}. \quad (\text{B.5})$$

2. Terms proportional to ϵ_{AB} , with two gamma matrices,

$$b D_\mu T_{\nu\rho}^- \gamma^{\rho 0} + T_{\nu\rho}^- T_{\mu\sigma}^+ \gamma^{\rho\sigma} - (\mu\nu) + \frac{g^2}{2} P^x \bar{P}^x \gamma_{\mu\nu} = -\frac{1}{4} R_{\mu\nu}{}^{ab} \gamma_{ab}. \quad (\text{B.6})$$

3. Terms proportional to σ_{AB}^x , no gamma matrix,

$$\begin{aligned} & \frac{g}{2} b \nabla_\mu P^x g_{\nu 0} - (\mu\nu) + g T_{\mu\nu}^- \bar{P}^x + g T_{\mu\nu}^+ P^x \\ & = g (L^\Lambda T_{\mu\nu}^+ - \bar{L}^\Lambda T_{\mu\nu}^- - 2i f_i^\Lambda G_{\mu\nu}^{i+} + 2i f_i^\Lambda G_{\mu\nu}^{i-}) P_\Lambda^x, \end{aligned} \quad (\text{B.7})$$

where we used that $-\Omega_{uv}^x \nabla_{[\mu} q^u \nabla_{\nu]} q^v = 0$, which follows from (4.17). Using $f_i^\Lambda P_\Lambda^x = 0$ from (4.14) we therefore find

$$\frac{g}{2} b \nabla_\mu P^x g_{\nu 0} - (\mu\nu) = -2g T_{\mu\nu}^- P_\Lambda^x \bar{L}^\Lambda. \quad (\text{B.8})$$

We now take components $\mu = \theta$ and use $\nabla_\theta P^x = 0$ and $g_{\theta 0} = 0$. We then find $T_{\theta\nu}^- P^x = 0$, whence $P^x = 0$ or $T_{\theta\nu}^- = 0$. In the latter case also $T_{\mu\nu}^- = 0$, because of the anti-self-duality property, and then $T_{\mu\nu} = 0$. We conclude

$$T_{\mu\nu}^- P_\Lambda^x L^\Lambda = 0. \quad (\text{B.9})$$

4. Terms proportional to σ_{AB}^x , two gamma. Using (B.9) we find

$$\epsilon^{xyz} P^y \overline{P^z} \gamma_{\mu\nu} = 0 . \quad (\text{B.10})$$

To summarize: we found two cases, one with $T_{\mu\nu}^- = 0$, the other with $P^x = 0$. We now list the remaining conditions for each case.

B.1 Case A: $T_{\mu\nu} = 0$

The remaining conditions are

$$\begin{aligned} \frac{g^2}{2} P^x \overline{P^x} \gamma_{\mu\nu} &= -\frac{1}{4} R_{\mu\nu}{}^{ab} \gamma_{ab} , \\ g_{i\bar{j}} \partial_{[\mu} z^i \partial_{\nu]} z^{\bar{j}} &= 0 , \\ \epsilon^{xyz} P^y \overline{P^z} &= 0 . \end{aligned} \quad (\text{B.11})$$

The first condition implies that the spacetime is maximally symmetric, with constant curvature $\propto P^x \overline{P^x}$. This case is discussed in section 4.6.1.

B.2 Case B: $P_\Lambda^x L^\Lambda = 0$

The remaining conditions are

$$\begin{aligned} b D_\mu T_{\nu 0}^- - (\mu\nu) &= -g_{i\bar{j}} \partial_{[\mu} z^i \partial_{\nu]} z^{\bar{j}} , \\ b D_\mu T_{\nu\rho}^- \gamma^{\rho 0} + T_{\nu\rho}^- T_{\mu\sigma}^+ \gamma^{\rho\sigma} - (\mu\nu) &= -\frac{1}{4} R_{\mu\nu}{}^{ab} \gamma_{ab} . \end{aligned} \quad (\text{B.12})$$

From the second condition we find the Riemann tensor

$$\begin{aligned} R_{\mu\nu\rho\sigma} &= R_{\mu\nu\rho\sigma}^- + R_{\mu\nu\rho\sigma}^+ , \\ R_{\mu\nu\rho\sigma}^- &= -b D_\mu T_{\nu\rho}^- e_\sigma^0 + T_{\nu\rho}^- T_{\mu\sigma}^+ - (\mu\nu) \\ &\quad - b D_\nu T_{\mu\sigma}^- e_\rho^0 + T_{\mu\sigma}^- T_{\nu\rho}^+ - (\mu\nu) \\ &\quad + b i \epsilon_{\rho\sigma}{}^{\lambda\kappa} D_\mu T_{\nu\lambda}^- e_\kappa^0 + i \epsilon_{\rho\sigma}{}^{\lambda\kappa} T_{\nu\lambda}^- T_{\mu\kappa}^+ - (\mu\nu) . \end{aligned}$$

This case is discussed in section 4.6.2.

C The Universal Hypermultiplet

The metric for the universal hypermultiplet is known to be

$$ds^2 = \frac{1}{R^2} \left(dR^2 + R(du^2 + dv^2) + \left(dD + \frac{1}{2}u dv - \frac{1}{2}v du \right)^2 \right) . \quad (C.1)$$

It describes the coset space $SU(2,1)/U(2)$ and therefore there are eight Killing vectors spanning the isometry group $SU(2,1)$. In the coordinates of (C.1), they can be written as

$$\begin{aligned} k_{a=1} &= \partial_D , \\ k_{a=2} &= \partial_u - \frac{v}{2} \partial_D , \\ k_{a=3} &= \partial_v + \frac{u}{2} \partial_D , \\ k_{a=4} &= -v \partial_u + u \partial_v , \\ k_{a=5} &= 2R \partial_R + u \partial_u + v \partial_v + 2D \partial_D , \\ k_{a=6} &= 2Rv \partial_R + 2(uv - D) \partial_u + (-2q + v^2 - u^2) \partial_v + (uq + Dv) \partial_D , \\ k_{a=7} &= 2Ru \partial_R + (-2q + u^2 - v^2) \partial_u + 2(D + uv) \partial_v + (-vq + Du) \partial_D , \\ k_{a=8} &= 2DR \partial_R + (Du - vq) \partial_u + (Dv + uq) \partial_v + (D^2 - q^2) \partial_D , \end{aligned} \quad (C.2)$$

where $q \equiv R + \frac{1}{4}(u^2 + v^2)$.

The moment maps are computed from

$$P^x = \Omega_{uv}^x D^u k^v . \quad (C.3)$$

The quaternionic two-forms Ω^x satisfy $\Omega^x \Omega^y = -\frac{1}{4} \delta^{xy} + \frac{1}{2} \epsilon^{xyz} \Omega^z$, and can be written as

$$\begin{aligned} \Omega^1 &= \frac{1}{2r^{3/2}} \left(dr \wedge du + dv \wedge dD + \frac{v}{2} du \wedge dv \right) , \\ \Omega^2 &= \frac{1}{2r^{3/2}} \left(-dr \wedge dv + du \wedge dD + \frac{u}{2} du \wedge dv \right) , \\ \Omega^3 &= \frac{1}{2r^2} \left(dr + dD - \frac{1}{2} v dr \wedge du + \frac{1}{2} u dr \wedge dv + r du \wedge dv \right) . \end{aligned} \quad (C.4)$$

We then find the moment maps

$$\begin{aligned}
P_{a=1} &= \left\{ 0, 0, -\frac{1}{2R} \right\}, \\
P_{a=2} &= \left\{ -\frac{1}{\sqrt{R}}, 0, \frac{v}{2R} \right\}, \\
P_{a=3} &= \left\{ 0, \frac{1}{\sqrt{R}}, -\frac{u}{2R} \right\}, \\
P_{a=4} &= \left\{ \frac{v}{\sqrt{R}}, \frac{u}{\sqrt{R}}, 1 - \frac{u^2 + v^2}{4R} \right\}, \\
P_{a=5} &= \left\{ -\frac{u}{\sqrt{R}}, \frac{v}{\sqrt{R}}, -\frac{D}{R} \right\}, \\
P_{a=6} &= \left\{ \frac{2(D - uv)}{\sqrt{R}}, \frac{2(q - u^2)}{\sqrt{R}}, \frac{-Dv - u(3q - u^2 - v^2)}{R} \right\}, \\
P_{a=7} &= \left\{ \frac{2(-q + v^2)}{\sqrt{R}}, \frac{2(D + uv)}{\sqrt{R}}, \frac{-Du + v(3q - u^2 - v^2)}{R} \right\}, \\
P_{a=8} &= \left\{ \frac{v(-4R + u^2 + v^2) - 4Du}{4\sqrt{R}}, \frac{-2qu + u^3 + 2Dv + uv^2}{2\sqrt{R}}, \frac{2R(u^2 + v^2) - q^2 - D^2}{2R} \right\}.
\end{aligned} \tag{C.5}$$

These formulae are needed for some of the examples that we consider in the main text of this paper.

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